

Visual* Learning of Arithmetic Operations

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* Visual: Picture In → Picture Out

Visual* Learning of Arithmetic Operations

Completely Impractical

Do we do any Practical Research
at The Hebrew University?

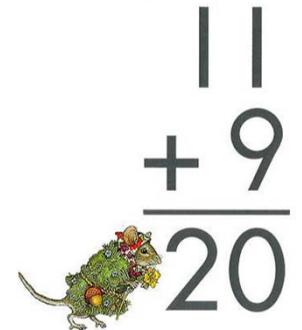
Shashua's MobilEye, US\$10B Company, Autonomous Driving

Peleg's Briefcam, doing
Video Synopsis (CVPR 2006)
2:30 min. video clip shown on
Taiwan TV 2 weeks ago

Yael Pritch, Alex Rav-Acha

Video Synopsis – Hebrew University





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Children Learn Arithmetic Visually

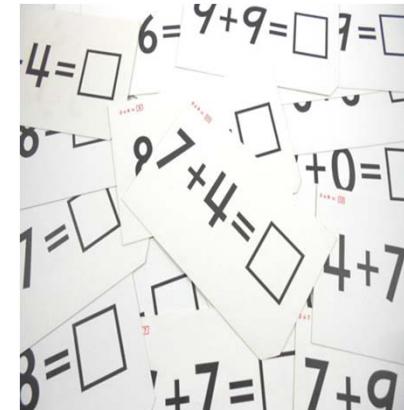
Numbers



Digits



Operations



Adults Perform Math Symbolically

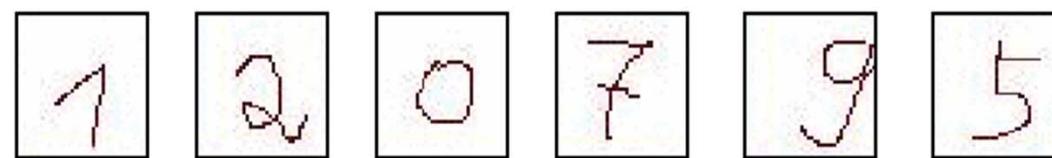
$$\begin{array}{r} 25 \\ +46 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 1 \\ 25 \\ +46 \\ \hline 71 \end{array}$$

- **Concept** of numbers
- **Digits** (0, 1, 5, 9)
- **Operations** (+, -, *, /)

Common Visual Machine Learning

Optical Character Recognition



Bitmap



Classifier

$$q: X \rightarrow Y$$



label

1 2 0 7 9 5

Can AI Methods Visually Learn Arithmetic?


$$\begin{array}{r} 11 \\ + 9 \\ \hline 20 \end{array}$$

Picture In
Picture Out

Visually means:

- *Create a picture of N_1*
- *Create a picture of N_2*
- *NN draws picture of $N_1 + N_2$*

981925

2010445

2992370

No Numbers, Digits, Operations

Traditional Arithmetic

Perception + Cognition + Action

Perception: OCR

9 \rightarrow 9

981925

Cognition:

2010445

$$981925 + 2010445 = 2992370$$

2992370

Action: Generate Picture 9 \rightarrow 9

2992370

Proposed NN: End-to-End Learning of All Tasks

Picture In \rightarrow Picture Out

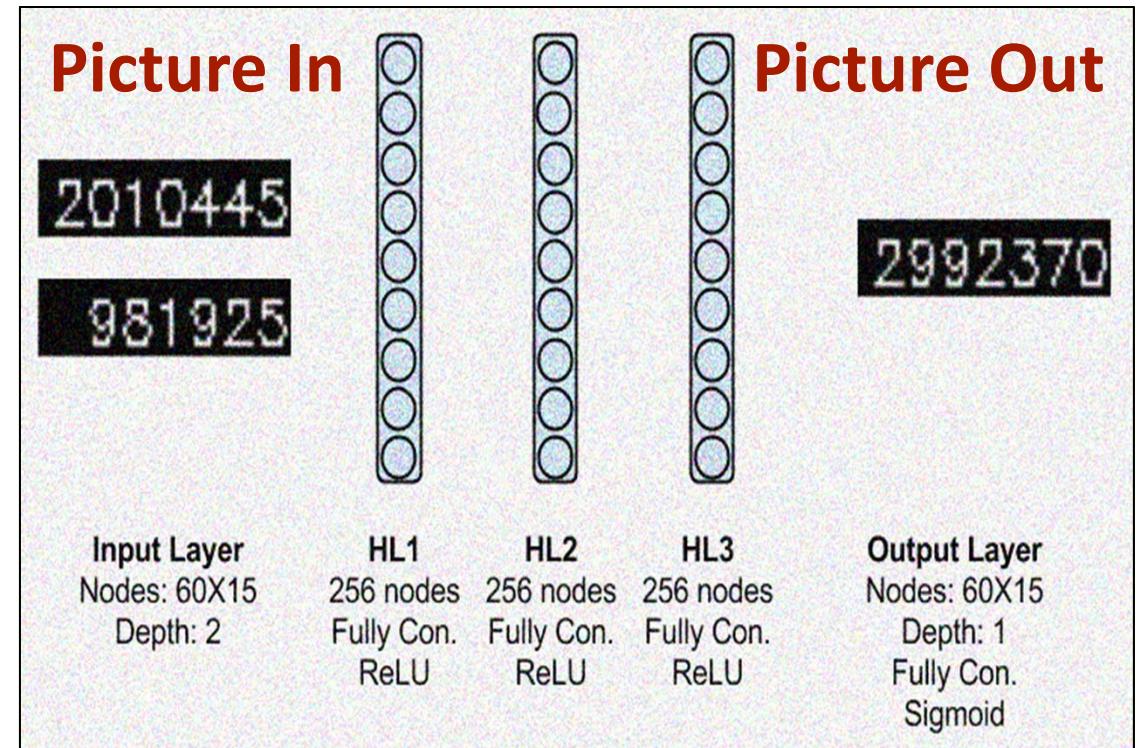
Visual Learning With Neural Networks

Input: Two pictures

- Each 60×15

Three Hidden layers:

- Affine 256 nodes
- ReLU



Output: One Picture (60×15), Sigmoid

Train with ADAM, alpha: = 0.1, mom: = 0.9, decay: 1e-4

Experiments

- Addition - range (0 .. 5M)
- Subtraction - range (0 .. 10M)
- Multiplication - range (0 .. 3.1K)

Training set: 150,000 examples (out of 25×10^{12})

Test set: 30,000 **unseen** examples

Example of Results

| | Addition | Subtraction | Addition + Noise |
|----------------|----------|-------------|------------------|
| Picture 1 | 981925 | 8184756 | 4752372 |
| Picture 2 | 2010445 | 5363281 | 230895 |
| NN Result | 2992370 | 2821475 | 4982677 |
| Correct Result | 2992370 | 2821475 | 4382677* |

Multiplication Failure Case

Picture 1

2531

Picture 2

1312

NN Result

3333332

Correct Result

3320672

Quantitative Results

| Operation | Visual end-to-end | | Cognition only: binary (1-hot to 1-hot) | |
|----------------|-------------------|---------------------|--|---------|
| | No. Layers | %OCR Digit Error | No. Layers | % Error |
| Addition | 3 | 1.9% | 1 | 1.7% |
| Subtraction | 3 | 3.2% | 1 | 2.1% |
| Multiplication | 5 | 71.5% | 3 | 37.6% |

- “+” & “-”: Both end-to-end & cognition succeeded¹⁴
- “*” : Both end-to-end & cognition failed

Implications

- Teach arithmetic to animals

- Poggio: Very few neurons
 - No language needed

- Explore more operations:

- Can visual learning be better than symbolic learning?

- Difficulty metric for exams:

- How hard is a psychometric exam
 - Invariance to language

