Supplement to Chapter 1 of *The Science of Digital Media* – Digital Data Representation and Communication

End-of-Chapter Exercises – Chapter 1 Digital Data Representation and Communication[1]

1. 600 cycles/s = 600 Hz

Angular frequency is 1200π radians/s

2. sin(1320π)

3. quantization error

4. 20,000 Hz

5. $2^{12}=4096$

6. false. The range of colors can be the same in that the extremes of colors can be equally different from each other. For example, with either eight or 16 bits, you can represent black at one extreme and white at the other, and then spread out the grayscale levels in between. The significance of bit depth is that with eight bits you'll have fewer shades of gray in between as opposed to the number of possible grayscale levels with 16 bits.

7. 4,276,012,000 bytes = 4,175,792.97 kB = 4077.92 MB = 3.98 GB

8. Applying the equation $20\log_{10}\left(\frac{2^{n-1}}{1/2}\right)=20\log_{10}(2^n)$, we get

$20\log_{10}\left(\frac{128}{1/2}\right)=20\log_{10}(256)$ = 20*2.4082 ≈ 48

9. (720 * 576) pixels/frame * 3 bytes/pixel * 25 frames/s * 60 s/m= 1,866,240,000 bytes/m of video

4 bytes/sample * 44100 samples/s * 60 s/m = 10,584,000 bytes/m

1,866,240,000 bytes/m + 10,584,000 bytes/m = 1,876,824,000 bytes/m

10. Applying the equation $D=2B\log_2(K)$, we have D = 2 *3 *$\log_2$(8) = 18 Mb/s.

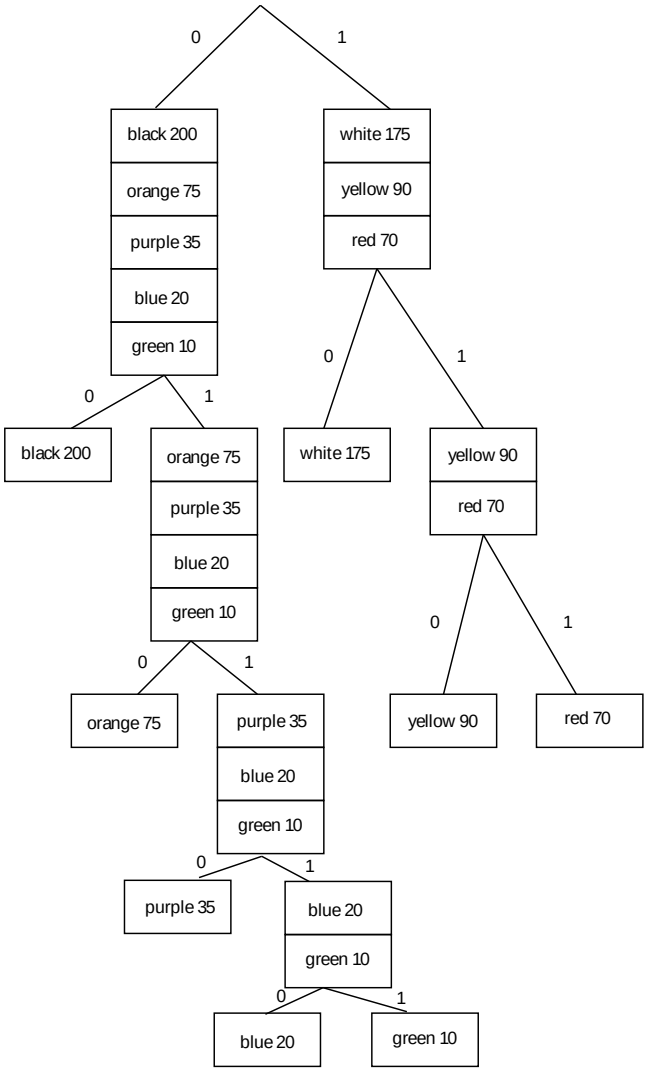
11. (240,7), (238,5), (230,4), (229,1), (228, 2), (227,21)

Assuming that the second number in each pair can be encoded in one byte, then we need 12 bytes for the compressed file. The uncompressed file requires 40 bytes. This is a compression rate of 40:12, which is equal to 10:3.

12.

[1]This material is based on work supported by the National Science Foundation under Grant No. DUE-0340969. This worksheet was written by Dr. Jennifer Burg, burg@wfu.edu.

color	frequency	optimum number of bits to encode this color	relative frequency of the color in the file	product of columns 3 and 4
black	100			
white	175			
yellow	70			
orange	75			
red	120			
purple	85			
blue	30			
green	20			
minimum average number of bits				



This gives the following codes

black 00

white 10

yellow 110

orange 010

red 111

purple 0110

blue 01110

green 01111

The average number of bits per pixel using this encoding is

(200 * 2 + 175 * 2 + 90 * 3 + 75 * 3 + 70 * 3 + 35 * 4 + 20 * 5 + 10 * 5)/675 = 2.585

This encoding gets pretty close to the minimum possible for the average bits per pixel.

13.

Range	Low value for probability interval	High value for probability interval	Symbol
1-0=1	0+1*0.4 = 0.4	0+1*0.7 = 0.7	White
0.7-0.4 = 0.3	0.4+0.3*0 = 0.4	0.4+0.3*0.4 = 0.52	Black
0.52-0.4 = 0.12	0.4+0.12*0.7 = 0.484	0.4+0.12*0.85 = 0.502	Yellow
0.502-0.484 = 0.018	0.484+0.018*0.95 = 0.5011	0.484+0.018*1 = 0.502	Blue
0.502-0.5011=0.0009	0.5011+0.0009*0.85=0.501865	0.5011+0.0009*0.95=0.501955	Red
0.501955-0.501865 = 0.00009	0.501865+0.00009*0.7 = 0.501928	0.501865+0.00009*0.85 = 0.5019415	Yellow